measured VSWR of both primary and secondary lines over the frequency band.

Conclusions

General synthesis procedures have been established for three-section and five-section symmetrical TEM-mode directional couplers. The synthesis leads to explicit formulas for the essential parameters, i.e., the normalized even-and odd-mode impedances, of three-section couplers. Although explicit formulas for the five-section couplers are not so readily obtainable, a sufficient amount of design information (in table form) is given for most practical coupler designs. An experimental model of a five-section coupler was built and tested, giving excellent agreement with theory.

Acknowledgment

The authors wish to thank R. D. Standley for his interesting and stimulating discussions, and R. S. Hollitch who programmed and ran the computer. Acknowledgment is also made to the IIT Research Institute for its financial support of this work.

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Theory and Tables of Optimum Symmetrical TEM-Mode Coupled-Transmission-Line Directional Couplers

E. G. CRISTAL, MEMBER, IEEE, AND L. YOUNG, SENIOR MEMBER, IEEE

Abstract—New equal-ripple polynomials were determined and applied to the synthesis of symmetrical TEM-mode coupled-transmission-line directional couplers (using exact methods). Tables of designs for symmetrical couplers of three, five, seven, and nine sections having mean couplings of −3.01, −6, −8.34, −10, and −20 dB, and having several equal-ripple tolerances in the coupling band are presented. Symmetrical maximally-flat directional-coupler designs having three, five, seven, and nine sections are also presented to complete the tables.

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The authors are with Stanford Research Institute, Menlo Park, Calif.

I. Introduction

A. General Properties of the Couplers

A SYMMETRICAL TEM-mode coupled-transmission-line directional coupler is shown schematically in Fig. 1. Note that the symmetrical directional coupler has symmetry with respect to two planes: Ports 1 and 2 have end-to-end symmetry with respect to Ports 3 and 4; Ports 2 and 3 have side-to-side symmetry with respect to Ports 1 and 4.

A TEM-mode coupled-transmission-line directional coupler, whether symmetrical or not, has the following properties, when a signal generator is connected to Port 1:

1) There is transfer of power from Port 1 to Port 2.
2) There is transfer of power from Port 1 to Port 4.
3) There is no transfer of power from Port 1 to Port 3.
4) There is no reflected wave out of Port 1.

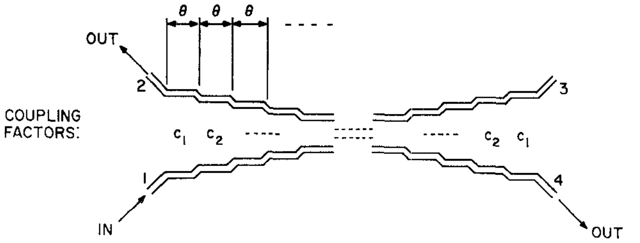

Fig. 1. Symmetrical TEM-mode coupled-transmission-line directional coupler.

The *symmetrical* directional coupler has, in addition, the unique and valuable property that

5) The two outputs at Ports 2 and 4 differ in phase by 90 degrees, at all frequencies.

It is this additional property that makes symmetrical couplers of particular importance.

Designs of optimum asymmetrical directional couplers of two to six sections were recently published by Levy [1]. These coupler designs have an equal-ripple approximation to the mean coupling. They are optimum in the sense that they provide a maximum bandwidth for a given number of sections, a given mean coupling, and a given coupling tolerance. To date, however, the exact design of optimum symmetrical couplers, i.e., symmetrical couplers having an equal-ripple approximation to the mean coupling, has been limited to couplers of at most three sections [2], [3]. Although symmetrical directional couplers of more than three sections can be synthesized on the basis of a first-order theory [4] these designs do not maximize the bandwidth since they do not necessarily provide an equal-ripple approximation to the mean coupling. Furthermore, for strong coupling, such as is required for 3-dB couplers, the first-order theory does not guarantee physically realizable results.

The first hurdle in the synthesis of optimum symmetrical couplers is to determine the appropriate equal-ripple polynomials for the insertion-loss function of the coupler. (These polynomials cannot be expressed in terms of known Chebyshev polynomials, as is the case for asymmetrical couplers.) Next, having obtained the insertion-loss function, extract the parameters of the coupler using exact synthesis procedures. Curves are plotted in Figs. 2, 3, and 4, showing typical responses of some of the five-, seven-, and nine-section 3-dB couplers that were obtained in this work.[1] These curves were calculated using the analytic expression for the coupling response rather than from an analysis of the synthesized couplers themselves.

[1] Whenever 3-db couplers are referred to, we shall mean equal power division, i.e., 3.0103-dB coupling.

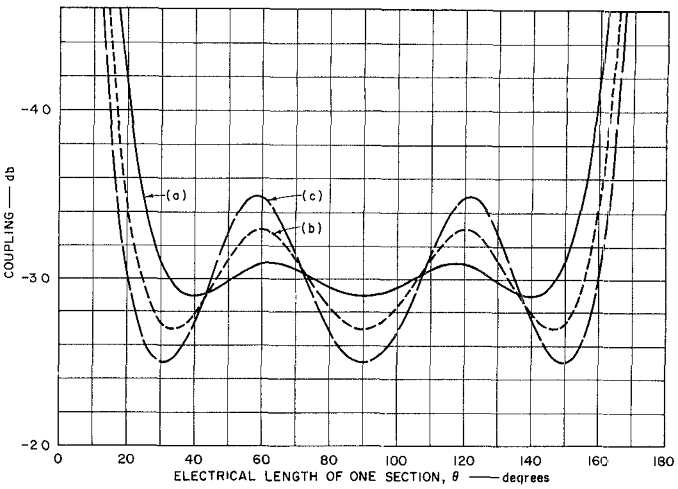

Fig. 2. Theoretical responses of symmetrical five-section couplers. (a) 0.1 dB ripple. (b) 0.3 dB ripple. (c) 0.5 dB ripple.

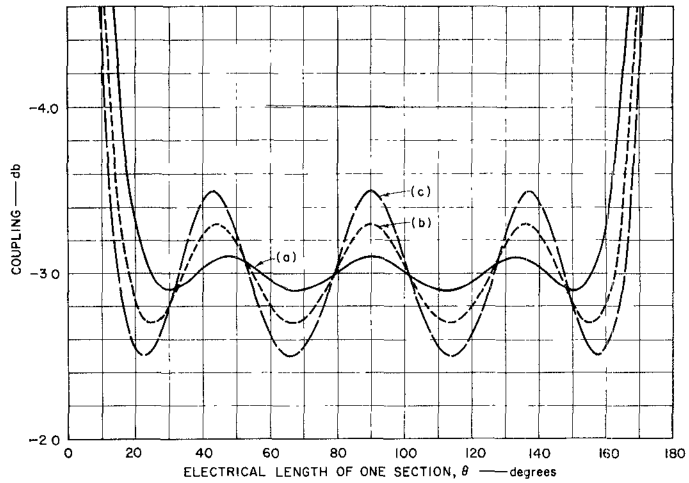

Fig. 3. Theoretical responses of symmetrical seven-section couplers, (a) 0.1 dB ripple. (b) 0.3 dB ripple. (c) 0.5 dB ripple.

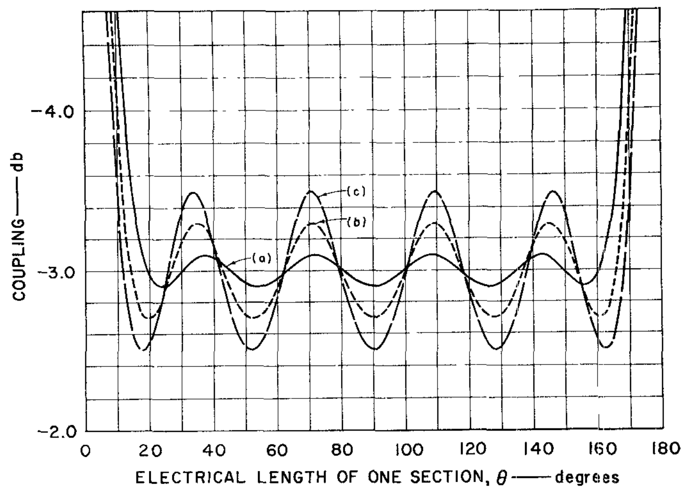

Fig. 4. Theoretical responses of symmetrical nine-section couplers. (a) 0.1 dB ripple. (b) 0.3 dB ripple. (c) 0.7 dB ripple.

B. Analytical Equivalence of Directional Couplers to Stepped-Impedance Filters

In papers by Feldshtein [5], Young [3], and Levy [6], an analytic equivalence is established between TEM-mode directional couplers and cascaded transmission lines. Briefly, the equivalence is that the reflected wave of the cascaded transmission lines corresponds to the backward-coupled wave of the directional coupler, and the transmitted wave of the cascaded transmission lines corresponds to the forward-coupled wave of the directional coupler. The use of this equivalence reduces the synthesis of TEM-mode coupled-transmission-line directional couplers having a prescribed coupling coefficient to the synthesis of cascaded transmission lines having a prescribed reflection coefficient.

C. The Insertion-Loss Function Theorem

Riblet [7] proved a theorem concerning the conditions that were necessary and sufficient if a given impedance function was to be realized as a cascade of equal-length transmission line sections. He also stated the most general insertion-loss function for a *quarter-wave transformer* (giving optimum match between two impedance levels). Levy [6] gave the most general insertion-loss function for the optimum asymmetrical quarter-wave filter (used as a prototype for the optimum asymmetrical TEM-mode coupler). Seidel and Rosen [8] have stated the necessary and sufficient conditions for making a prescribed insertion-loss function realizable as a cascade of equal-length transmission-line sections. They did for the insertion-loss function what Riblet did for the impedance function, and they arrived at a more concise statement. We shall state Seidel and Rosen's theorem as follows:

The necessary and sufficient conditions that an insertion-loss function L represents a homogeneous[2] stepped-impedance filter is that it be a polynomial of the form

$$L = L(\sin^2 \theta) \tag{1}$$

and that L be a polynomial greater than or equal to unity for all real values of θ.

D. The Symmetry Condition

The symmetry of a two-port filter is closely related to its phase properties. It can readily be shown (by an extension of the argument in [9], for instance) that the necessary and sufficient condition for a two-port filter to be symmetrical is that the phase of the transmission coefficient and the phase of the reflection coefficient differ by 90 degrees. This is the property that makes the symmetrical coupler so interesting and useful.

It is shown in textbooks on network synthesis [10], [11] that the necessary and sufficient condition for having an insertion-loss function represent a (lumped-constant) symmetrical network is that the function have the form: Unity plus the square of an odd function of frequency. By Richards' transformation [12], [13] we can now extend this result to resistor transmission-line circuits, including the stepped-impedance filters under present consideration. When this is done, it is found that the insertion-loss function is of the form: Unity plus the square of an odd function of θ. Combining this result with the insertion-loss theorem, we conclude that:

The necessary and sufficient condition that an insertion-loss function L represent[3] a *symmetical* homogeneous stepped-impedance filter, of n equal-line-length sections, is that it be of the form [21]

$$L = 1 + [P_n(\sin \theta)]^2 \tag{2}$$

where P_n is an odd polynomial in $\sin \theta$, of degree n.

Thus, the synthesis of a symmetrical directional coupler reduces to:

1) Finding the optimum odd polynomials P_n $(\sin \theta)$ and
3) Extracting the transmission-line impedances from the resulting insertion-loss function.

The synthesis procedure just outlined can produce many coupler designs, the number depending on the degree n of $P_n(\sin \theta)$. To obtain a symmetrical design, it is necessary to select the complex zeros of the reflection coefficient (obtained analytically from the insertion-loss function) such that symmetry with respect to the j-axis in the complex plane is achieved. This will be discussed further in Section IV.

II. Tables of Symmetrical Couplers

Tables of equal-ripple and maximally-flat symmetrical coupler designs are presented in the Appendix. TEM-mode coupled-transmission-line directional couplers are conveniently analyzed on the basis of the even- and odd-mode impedances of the individual sections of the coupler [14]. The tables of coupler designs presented here are, therefore, given on this basis. In addition, the signs have been normalized so that the product of even- and odd-mode impedances is unity; that is,

$$Z_{oe}Z_{oo} = 1 \tag{3}$$

where Z_{oe} and Z_{oo} are the even- and odd-mode impedances, respectively. For any particular application, the even- and odd-mode impedances are scaled by multiplying each normalized impedance of the tables by the value of the impedance of the terminating line. Since Z_{oo} may be obtained from (3), only Z_{oe} is tabu-

[2] By *homogeneous* we mean that the ratios of the impedances of the cascaded lines, and the ratios of the wavelengths in them, be independent of frequency, *cf.* [20].

[3] At the same time, it should be pointed out that an insertion-loss function of the stated form can be represented by a number of networks, and this theorem states that at least one of them is symmetrical.

lated. To keep the tables as compact as possible, only one-half of the even-mode impedances for each design are presented, since the couplers are symmetrical.

A parameter of frequent interest to the designer is c_i, the coefficient of coupling of the ith section of the coupler. Because (3) is normalized, the coefficient of coupling can be obtained from the values of Z_{oe} in the tables by the formula

$$c_i = \frac{(Z_{oe})_i^2 - 1}{(Z_{oe})_i^2 + 1}. \tag{4}$$

Two bandwidth definitions for directional couplers are in common use, and both are presented in the tables for the convenience of the reader. The *fractional bandwidth*, denoted as w is given by

$$w = \frac{f_2 - f_1}{f_0} \tag{5}$$

where f_1 and f_2 are the lower and upper frequencies at the equal-ripple band edge (see Fig. 6), and

$$f_0 = \frac{f_2 + f_1}{2} \tag{6}$$

is the arithmetic mean of f_1 and f_2.

The second definition of bandwidth is the *bandwidth ratio*, denoted here by B; it is given by

$$B = \frac{f_2}{f_1} \tag{7}$$

where f_2 and f_1 are defined as before.

In the cases of maximally-flat coupler designs (Tables A-21 to A-24) the frequencies f_2 and f_1 used in the formulas for bandwidth refer to the frequencies that are 3 dB lower than the mean coupling.

The parameters M and δ that appear in Tables A-1 through A-20 in the Appendix, pp. 554–557, are explained by the representative coupling curve shown in Fig. 5. The symbol M represents the mean coupling in decibels. The symbol δ represents the maximum deviation (ripple value) from the mean in decibels. All designs presented in the tables have each coupling section one-quarter wavelength long at the mean frequency f_0.

Symmetrical coupler designs of three sections are included in the tables, although these designs may be obtained from prior work by Shimizu and Jones [2] or Young [3]. The design values in the tables were arrived at independently by the methods given in Section III. They are included only to make the tables as complete and useful as possible. Maximally-flat coupler designs are also presented in the tables. Exact designs of the three-section maximally-flat coupler have been published previously [3], but the design is included here. The designs for maximally flat couplers of 5, 7, and 9 sections are believed to be new.

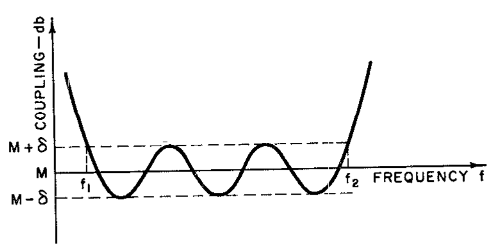

Fig. 5. Typical directional coupler characteristics (the particular curve shown represents a five-section coupler).

III. Deviation of the Tables

The derivation of the coupler parameters from the given insertion-loss function is presented here. (The derivation of the insertion-loss functions themselves will be presented in Section IV.) The synthesis of stepped-impedance lines from a prescribed insertion-loss function has been adequately described in previous papers [6], [7]. For this reason the synthesis description here is presented as a step-by-step procedure only, and is stated without proof.

Let $P_n(x)$ be an odd polynomial in x of degree n that makes the function

$$L(x) = 1 + P_n^2(x) \tag{8}$$

an equal-ripple function on the interval 0 to 1.

Letting $x = \sin\theta$, the function $L(x)$ may be identified as an equal-ripple insertion-loss function for a symmetrical cascade of transmission lines.

From (8), the magnitude squared of the reflection coefficient is obtained:

$$|\Gamma|^2 = \frac{P_n^2(x)}{1 + P_n^2(x)} \tag{9}$$

where Γ is the reflection coefficient, and again $x = \sin\theta$.

Next, using Richards' transformation in the form [12]

$$\tan\theta = s/j, \tag{10}$$

where s is a complex variable, and $j = \sqrt{-1}$, the substitution

$$x = \frac{s/j}{\sqrt{1 + (s/j)^2}} = \sin\theta \tag{11}$$

may be made for the variable x. To facilitate the computations which follow, however, the transformation (11) is accomplished in two parts:

Part 1) Replace x by $\hat{s}/j$, where in this part $\hat{s}$ is an intermediate variable and not equal to $j\tan\theta$ (12)

Part 2) Replace $\hat{s}/j$ by $(s/j)/\sqrt{1+(s/j)^2}$ (13)

Equation (13) completes the transformation of (11). Using the previous two-part transformation process, the synthesis procedure is as follows:

Step A: Transformation (12) is applied to (9) and the right side of (9) is factored into the form

$$\bar{\Gamma}(\hat{s})\bar{\Gamma}(-\hat{s}) = \Gamma(\hat{s}/j)\Gamma(-\hat{s}/j) \propto \frac{\prod_{i=1}^{n} (\hat{s} - z_i)(-\hat{s} - z_i)}{\prod_{i=1}^{n} (\hat{s} - p_i)(-\hat{s} - p_i)} \tag{14}$$

where $\bar{\Gamma}(\hat{s})$ is defined to be $\Gamma(\hat{s}/j)$, z_i is a zero of the numerator, and p_i is a zero of the denominator. Expression (14) is not stated as an equation because the necessary constant factors in the numerator and denominator are excluded at this point. The factoring of (9) is simplified by solving the lower-order equations

$$P_n(\hat{s}/j) = 0 \tag{15}$$

and

$$P_n(\hat{s}/j) = \pm j \tag{16}$$

rather than (9) itself. It is clear that the zeros of (15) are to be taken as double.

It can be demonstrated that where $P_n(x)$ is an odd polynomial, the values of s that are solutions to

$$P_n(\hat{s}/j) = j \tag{17}$$

are the negative of the values of $\hat{s}$ that satisfy

$$P_n(\hat{s}/j) = -j. \tag{18}$$

so that it is only necessary to solve one of the equations of (16).

Step B: Next, the zeros of the numerator and denominator are mapped into new zeros, denoted by primes, by the transformation of (13). This is equivalent to

$$z_i' = \frac{z_i}{\sqrt{1 + (z_i)^2}} \tag{19}$$

$$p_i' = \frac{p_i}{\sqrt{1 + (p_i)^2}} \cdot \tag{20}$$

The reflection coefficient $\Gamma'(s)$ is then constructed in the following way: Zeros of the numerator of $\Gamma'(s)$ are chosen from the z' so that they are symmetrical with respect to the j axis. This is necessary to ensure that the network will be symmetrical. The zeros of the denominator of $\Gamma'(s)$ are chosen from the p' so they lie in the left half plane. This latter selection process is necessary to ensure that the reflection coefficient is analytic in the right half plane. (Network symmetry will be discussed in more detail subsequently.) After the zero and pole selection is completed, the complex reflection coefficient is constructed according to (21),

$$\Gamma'(s) = \frac{\gamma_N'(s)}{\gamma_D'(s)} = \frac{a \prod_{i=1}^{n} (s - z_i')}{b \prod_{i=1}^{n} (s - p_i')} \tag{21}$$

where $\gamma_N'(s)$ and $\gamma_D'(s)$ are equal to the numerator and denominator, respectively, of the right side of (21).

Step C: The constants a and b in (21) are evaluated as follows: An examination of (11) reveals that the point $\sqrt{2}$ goes into $\sqrt{2}$. Furthermore, the transformation (11) also requires that

$$P_n^2(\sqrt{2}) = a^2 \left. \frac{\prod_{i=1}^{n} (s - z_i') \prod^{n} (-s - z_i')}{(-1)^n} \right|_{s=\sqrt{2}} \cdot \tag{22}$$

Solving gives

$$a = \left\{ \frac{(-1)^n P_n^2(\sqrt{2})}{\prod_{i=1}^{n} (s - z_i') \prod_{i=1}^{n} (-s - z_i')} \right\}_{s=\sqrt{2}}^{1/2} \cdot \tag{23}$$

The sign of a is chosen to that $\gamma_N'(s) \to +0$ as $s \to 0$. This choice of sign is justified by noting that the normalized even-mode impedances are always greater than one, which requires the complex reflection coefficient to be positive as $s \to 0$. Similarly, the constant b may be found from:

$$b = \left\{ \frac{(-1)^n [1 + P_n^2(\sqrt{2})]}{\prod_{i=1}^{n} (s - p_i') \prod_{i=1}^{n} (-s - p_i')} \right\}_{s=\sqrt{2}}^{1/2} \cdot \tag{24}$$

Step D: When

$$\Gamma'(s) = \frac{\gamma_N'(s)}{\gamma_D'(s)}$$

has been obtained, the impedance function is determined from the relationship

$$Z(s) = \frac{1 + \Gamma'}{1 - \Gamma'} = \frac{\gamma_D' + \gamma_N}{\gamma_D' - \gamma_N'} \cdot \tag{25}$$

From Z, the *ABCD* transmission matrix is constructed[4] as follows:

A is identified with the even part of $\gamma_D' + \gamma_N'$,

B is identified with the odd part of $\gamma_D' + \gamma_N'$,

C is identified with the odd part of $\gamma_D' - \gamma_N'$,

D is identified with the even part of $\gamma_D' - \gamma_N'$. (26)

It can now be seen why the zeros of $\Gamma'(s)$ must be chosen symmetrical with respect to the j axis. For a network to be symmetrical it is necessary that its *ABCD* matrix have $A = D$. From (26) it is seen that A is the even part of $\gamma_D' + \gamma_N'$ while D is the even part of $\gamma_D' - \gamma_N'$. The only way to ensure that $A = D$ is to make γ_N' an odd polynomial. Choosing the zeros of the numerator of $\Gamma'(s)$ to be symmetrical with respect to the j axis (and recalling that there is a zero in the origin) forces γ_N' to

[4] A constant premultiplier of the *ABCD* matrix, $(1/\sqrt{1-s^2})^n$, is neglected in the construction process.

be an odd polynomial. Other selections of the zeros of the numerator $\Gamma'(s)$ will result in asymmetrical structures, although the same insertion-loss function will result.

Step E: Next, the transmission-line impedances are extracted from the $ABCD$ matrix.

The first line impedance is given by the formula

$$Z_1 = \left.\frac{A(s)}{C(s)}\right|_{s=1} = \left.\frac{B(s)}{D(s)}\right|_{s=1}. \tag{27}$$

Next, the matrix multiplication

$$\frac{1}{1-s^2}\begin{bmatrix} 1 & -Z_1 s \\ \frac{s}{-Z_1} & 1 \end{bmatrix}\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \overline{A} & \overline{B} \\ \overline{C} & \overline{D} \end{bmatrix} \tag{28}$$

is performed. The result is a new $ABCD$ matrix, which has the same form as the preceding $ABCD$ matrix, except that the new input impedance

$$\overline{Z}(s) = \frac{\overline{A}+\overline{B}}{\overline{C}+\overline{D}} \tag{29}$$

is of one degree less than the preceding $Z(s)$. The next line impedance is given by (27),

$$Z_2 = \left.\frac{\overline{A}(s)}{\overline{C}(s)}\right|_{s=1} \tag{30}$$

and the reduction process of (28) is repeated.

In this way all the line impedances may be obtained. Since the structure is symmetrical, however, it is only necessary to perform the cycle a total of $(n+1)/2$ times where n is the (odd) number of sections in the line.

Example: A numerical example will serve to illustrate this synthesis procedure. The insertion-loss function of a three-section maximally-flat symmetrical coupler with equal-power division at $x=\sin\theta=1$ can be shown to be[5]

$$\begin{aligned} L &= 1 + (1.5x - 0.5x^3)^2 \\ &= 1 + P_3^2(x). \end{aligned} \tag{31}$$

Using *Step A*, we find that

$$P_3(\hat{s}/j) = 0 \tag{32}$$

has double roots

$$z = 0, \qquad j1.732, \qquad -j1.732 \tag{33}$$

and that

$$P_3(\hat{s}/j) = j \tag{34}$$

has roots

$$p = -0.5961, \qquad (0.2980 \pm j1.8073). \tag{35}$$

[5] The general form of maximally-flat odd polynomials is presented in Part IV.

Therefore, the roots of $P_3(\hat{s}/j) = -j$ are

$$p = +0.5961, \qquad -(0.2980 \pm j1.8073). \tag{36}$$

Using *Step B* next, it is found that the roots are transformed into

$$z' = 0, \qquad \pm(1.2247) \tag{37}$$

and

$$p' = \pm 0.5120, \qquad \pm(1.1726 \pm j0.07782). \tag{38}$$

From *Step C*,

$$\begin{aligned} &a \text{ is found to be } 1.0 \\ &b \text{ is found to be } 1.414. \end{aligned} \tag{39}$$

From *Step D*,

$$\begin{aligned} A(s) &= 1.0 + 4.0407s^2 \\ B(s) &= 5.1512s + 0.41421s^3 \\ C(s) &= 2.1512s + 2.4142s^3 \\ D(s) &= 1.0 + 4.0407s^2. \end{aligned} \tag{40}$$

Last, the impedances are extracted as explained in *Step E*. The results are

$$Z_1 = \frac{A(1)}{C(1)} = \frac{5.0407}{4.5657} = 1.104 \tag{41}$$

which is also equal to Z_3,

Next, Z_2 is found to be

$$Z_2 = \frac{\overline{A}(1)}{\overline{C}(1)} = 2.943. \tag{42}$$

IV. Derivation of the Equal-Ripple and Maximally-Flat Polynomials

A. Equal-Ripple Polynomials

The determination of the equal-ripple polynomials used in the insertion-loss functions of the symmetrical couplers is described here. This problem may be viewed as the determination of odd polynomials of degree n that give an equal-ripple approximation to a constant on the interval of zero to one. The polynomials take their last "equal-ripple value" at $x=1$.

Consider for an example the case of a fifth-order odd polynomial approximating unity on $0<x\leq 1$. This is shown in Fig. 6. It is clear from the figure that there are two points at thich the first derivative of the polynomial is zero. Let these points be denoted as x_1 and x_2. Because the polynomial is odd, $P_5'(x)$ (where the prime denotes d/dx) may be written as

$$P_5' = C(x^2 - x_1^2)(x^2 - x_2^2). \tag{43}$$

From (43), $P_5(x)$ is determined by integration:

$$P_5(x) = C\int_0^x (u^2 - x_1^2)(u^2 - x_2^2)\,du. \tag{44}$$

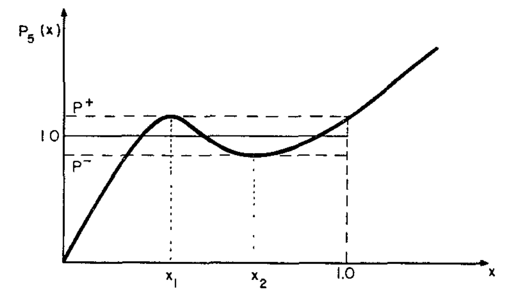

Fig. 6. Example of a fifth-order odd polynomial approximating unity on the interval zero to one.

The constant C is determined by the condition that

$$P_5(1) = P^+ \text{(see Fig. 7 for definition of } P^+). \quad (45)$$

The result is

$$P_5(x) = c_1(x_1, x_2)x + c_3(x_1, x_2)x^3 + c_5(x_1, x_2)x^5. \quad (46)$$

The coefficients c_i are determined by the conditions that

$$P_5(x_1) = P^+$$
$$P_5(x_2) = P^- \quad (47)$$

where P^+ and P^- are the equal-ripple extremes (see Fig. 7).

Before describing the solution of (47), we wish to generalize the procedure just given. For an nth-order odd polynomial approximating a constant on the interval zero to one, we have

$$P_n'(x) = C \prod_{i=1}^{k} (x^2 - x_i^2) \quad (48)$$

where

$$k = \frac{n-1}{2} \quad (49)$$

$$P_n(x) = C \int_0^x \prod_{i=1}^{k} (u^2 - x_i^2) du \quad (50)$$

and

$$C = \frac{P_n(1)}{\int_0^1 \prod_{i=1}^{k} (u^2 - x_i^2) du} \quad (51)$$

where $P_n(1)$ equals P^+ or P^- depending on the value of n. The result of the preceding operations is an expression for $P_n(x)$ in the form

$$P_n(x) = \sum_{i=1}^{n} c_i x^i \quad (52)$$

where c_i are functions of the x_i and all coefficients with even i are zero. The c_i are determined by the condition that

$$P_n(x, x_1, x_2, \cdots, x_k)\big|_{x=x_1} = P^+$$
$$P_n(x, x_1, x_2, \cdots, x_k)\big|_{x=x_2} = P^-$$
$$\vdots$$
$$P_n(x, x_1, x_2, \cdots, x_k)\big|_{x=x_k} = P^+ \text{ or } P^-$$
$$\text{depending on } n. \quad (53)$$

The following method was used to solve the set of nonlinear equations in (53). Considering P_n as a multivariable function of the x_i each equation of the set was linearized by taking the first two terms of the generalized Taylor expansion of the functions on the left in (53). Initial guesses for the values of $x_1, x_2, \cdots, x_k$ were substituted into the resultant linear set, and a solution was obtained by standard methods. The new solution was used as a second approximation to the initial guess and the process was repeated. (This method is equivalent to Newton's method for a single variable but is generalized here to the $(n-1)/2$ variable case [15].) The linear set of equations takes the form

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1,k} \\ a_{21} & a_{22} & \cdots & a_{2,k} \\ \vdots & & & \\ a_{k,1} & & \cdots & a_{k,k} \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \vdots \\ \Delta x_k \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_k \end{bmatrix} \quad (54)$$

where

$$k = \frac{n-1}{2},$$

Δx_i is the correction to previous x_i,
$b_i = P^+ - P(x_i)$ or $P^- - P(x_i)$, whichever applies.

Also, for $i \neq j$ and for $i, j = 1, 2, 3 \cdots k$

$$a_{ij} = \sum_{n=1}^{k} \frac{\partial c_n}{\partial x_j} x_i^n, \qquad (n \text{ odd}) \quad (55)$$

and for $i = 1, 2 \cdots k$

$$a_{ii} = \sum_{n=1}^{k} \left\{ \frac{\partial c_n}{\partial x_i} x_i^n + n c_n x_i^{n-1} \right\}, \qquad (n \text{ odd}). \quad (56)$$

This iterative method of solution was found to be a very rapidly convergent process. In a few cases where the initial guesses were not close to the solutions, the iterative process still converged but gave some x_i outside the interval zero to one.[6] However, these solutions were scaled to the interval zero to one to obtain the sought-for answers.

For cases where the polynomials are to approximate a constant value M, P^+ may be defined as $M + \delta$ where δ is the maximum equal-ripple deviation; P^- may then be defined as $M - \delta$ which gives M as the arithmetic

[6] A consideration of the set (53) of equations shows that there exist multiple solutions outside the interval zero to one, depending on the value of n.

Table 1

COEFFICIENTS FOR EQUAL-RIPPLE POLYNOMIALS APPROXIMATING UNITY ON THE INTERVAL ZERO TO ONE

δ	C_1	C_3	C_5	C_7	C_9
0.1	2.1952143	- 1.2952143			
0.1	3.4118700	- 5.7150991	3.4032291		
0.1	4.6700175	-15.3542579	22.6907481	- 11.106507	
0.1	5.94547862	-32.3085509	85.6573300	- 97.6641647	39.4699070
0.2	2.5923814	- 1.7923814			
0.2	4.1112287	- 8.2285005	5.3172718		
0.2	5.66731189	-22.4205945	36.0524481	- 18.4991655	
0.2	7.238206	-47.4794411	137.138390	-164.008299	68.3111441

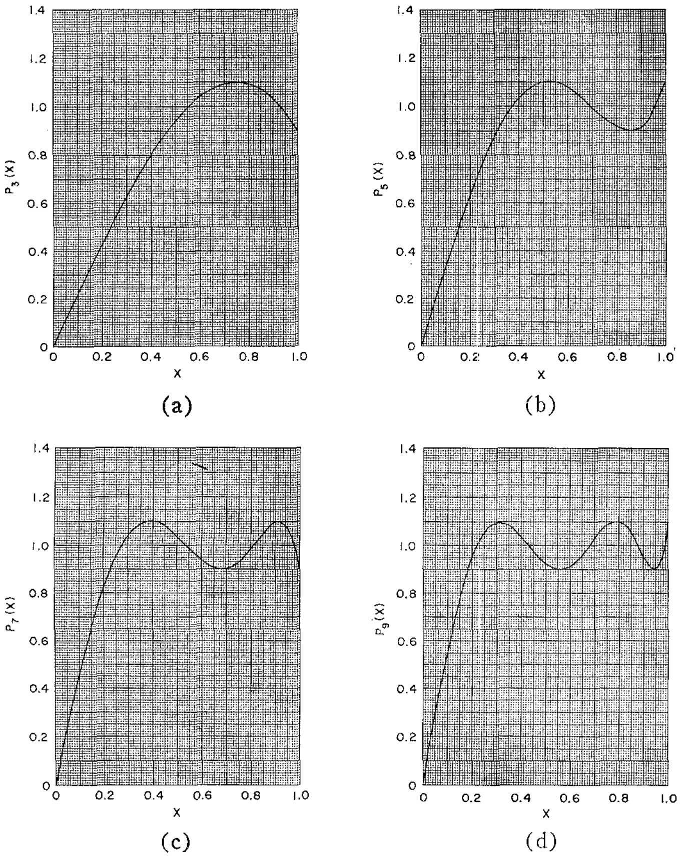

Fig. 7. Equal-ripple approximations to unity on the interval zero to one (equal-ripple tolerance 0.1). (a) Third-order polynomial. (b) Fifth-order polynomial. (c) Seventh-order polynomial. (d) Ninth-order polynomial.

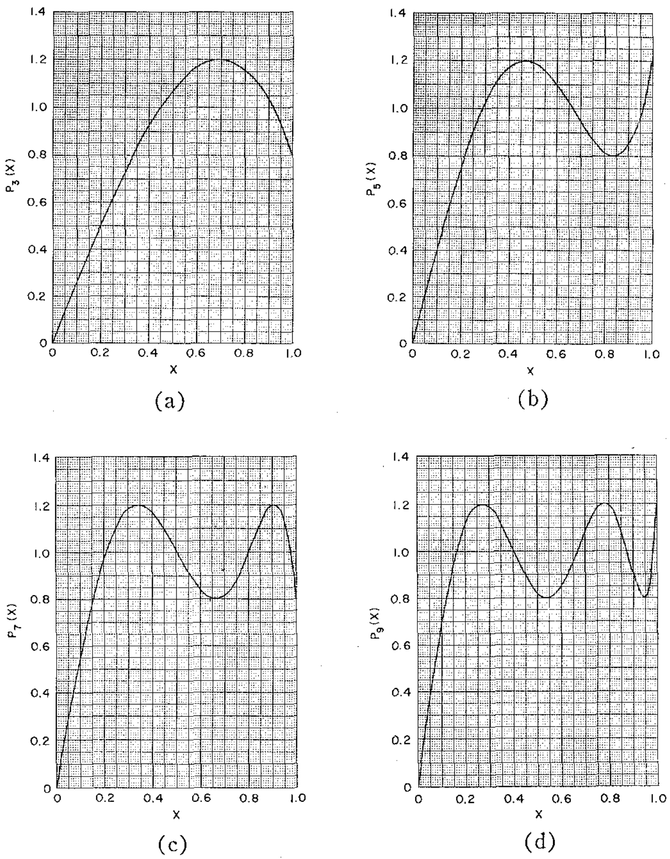

Fig. 8. Equal-ripple approximations to unity on the interval zero to one (equal-ripple tolerance 0.2). (a) Third-order polynomial. (b) Fifth-order polynomial. (c) Seventh-order polynomial. (d) Ninth-order polynomial.

mean. Another possibility is to define P^- as $1/(M+\delta)$ which gives M as the geometric mean. For the cases involving the synthesis of the symmetric couplers, it was required that the coupling in decibels be an equal-ripple function. For these cases the values of P^+ and P^- were defined as

$$P^+ = \left\{\frac{1}{\dfrac{A(M)}{A(\delta)} - 1}\right\}^{1/2}$$

$$P^- = \left\{\frac{1}{A(M)A(\delta) - 1}\right\}^{1/2} \qquad (57)$$

where the function $A(x)$ is defined as

$A(x) = \text{antilog}_{10}(x/10)$.

M is the mean coupling value in decibels.

δ is the maximum deviation in decibels from the mean coupling.

To illustrate a result of the previously described method of obtaining equal-ripple polynomials, several representative polynomials which approximate unity on the interval zero to one are tabulated in Table 1, and shown in Figs. 7 and 8. The polynomials in Table 1 approximate unity in an arithmetic-mean sense: that is, $P^+ = 1+\delta$ and $P^- = 1-\delta$. The case for $\delta = 0.1$ and 0.2 are shown in Figs. 7 and 8, respectively. [Table 1 was not used to synthesize couplers, since another equal-ripple criterion was used, as previously explained, namely (57) and (58).]

B. *Maximally-Flat Polynomials*

The derivation of the maximally-flat polynomials is given in this part. A suitable starting point is (48) where all s_i are set equal to unity. That is,

$$P_n'(x) = C(x^2 - 1)^k. \tag{62}$$

Integrating (62) gives

$$P_n(x) = C\int_0^x (u^2 - 1)^k du \tag{63}$$

and C is evaluated by

$$C = \frac{P_n(1)}{\int_0^1 (u^2 - 1)^k du}. \tag{64}$$

For the maximally-flat polynomials, $P_n(1)$ equals either (57) or (58) with δ set equal to zero.

V. Discussion

A. *On 3.01-dB Directional Couplers as Equal Power Splitters*

In Section V the derivation of the equal-ripple polynomials was described, and several possible definitions of P^+ and P^- (the equal-ripple extremes) were stated. The particular definition that is most appropriate will depend on the application. For the synthesis of symmetrical couplers, (57) and (58) were used for P^+ and P^-. These definitions give an equal-ripple approximation of the mean coupling in decibels, in an arithmetic sense. Since in applications involving couplers, specifications are usually stated in terms of decibels, these definitions are certainly appropriate. However, in many applications involving 3-dB couplers,[7] what is usually desired is an equal power splitter. A specification like 3 ± 0.3 dB (for example) is, therefore, not strictly appropriate since it cannot apply to both output ports simultaneously. Thus, 3 ± 0.3 dB corresponds to a coupling coefficient having squared magnitude between 0.4677 and 0.5370, which gives an equal-ripple approximation (in the arithmetic mean for coupled power) to an average of 0.5023 rather than 0.5000. The forward power varies from 0.5323 to 0.4630, having an arithmetic mean value of 0.4977. As the ripple deviation in decibels gets larger, the arithmetic mean of the coupling coefficient becomes even further from 0.5. In view of this problem, consideration was given to designing the "3-dB hybrids" on the basis that the squared magnitude of the coupling coefficient be $0.5000\pm\delta$, so that the power division out of both ports would be equal-ripple in the same way. It was finally decided to stick to the conventional approach, and to synthesize on the basis of (57) and (58). The difference in any practical problem is small. In addition, the synthesis on the basis of equal-ripple coupling in decibels, i.e., on a log basis, is in keeping with previously published work, and is also more appropriate for other-than-3-dB couplings.

[7] The designation 3-dB coupler is intended to imply equal-power division. A more correct designation would be 3.0103-dB coupler.

B. *Applications of Symmetrical 3-dB Couplers*

Directional couplers with 3-dB coupling are power dividers in which the power is evenly divided between the two outputs. In general, there need not be any particular phase relationship between the outputs. If the two output arms are short-circuited or open-circuited in two positions where the emerging waves are 90 degrees out of phase, then the reflected waves add up in the remaining (fourth) arm, and no power is reflected into the input arm. This property is very useful for some applications, in many of which a circulator could also have been used. Much greater bandwidths can be realized with TEM-mode directional couplers than with circulators, and the couplers can operate in any frequency band. Other typical advantages may include higher isolation and better input VSWR. Applications for which symmetrical couplers are best suited include diplexers and multiplexers, directional filters, phase shifters, balanced mixers, duplexers, negative-resistance amplifiers, and others in which the 90-degree phase-difference property is essential. It is only in *symmetrical* couplers that two positions can be found in the output arms where the 90-degree phase difference is maintained independent of frequency. A further practical advantage of symmetrical couplers is that the strongest coupling region is in the center and not at one end [1], [6] so that it becomes less difficult to connect to all four ports.

C. *Other Designs Realizing the Same Insertion-Loss Function*

In the synthesis procedure described in Section III, the magnitude squared of the reflection coefficient for real frequencies is to be generalized (by analytic continuation) to

$$|\Gamma|^2 \to \Gamma(s)\Gamma(-s).$$

If the symmetrical coupler is to be realized, the numerator of $\Gamma(s)$ must have zeros that are symmetrical with respect to the j axis. However, unsymmetrical couplers can be obtained from the insertion-loss function by selecting other permissible choices for the zeros of $\Gamma(s)$. For example, Levy chose all zeros of his $\Gamma(s)$ to be in the left half plane, and obtained couplers having monotonically increasing Z_{oe} within the coupling region. By choosing other combinations of zeros, placing some zeros in the left half plane and others in the right half plane, realizations are possible that have Z_{oe} neither monotonically increasing nor symmetrical. The maximum number of possible realizations will depend on the degree n of the insertion-loss polynomial.

D. *Physical Realization of the Couplers*

This paper is concerned only with the design of the circuit parameters of symmetrical couplers. For the realization of the final physical dimensions, the reader is referred to the extensive literature on the subject. He will find a good account with many references up to 1963 in the book by Matthaei Young, and Jones [4]. For weak to moderate coupling, coupler designs may be realized with coupled round rods. Design data of Honey [14] are appropriate for weak coupling, while the data of Cristal [16] are more accurate for stronger coupling, although they may be used for weak coupling also. For moderate to tight coupling, Getsinger [17] presents data on rectangular bars, and Cohn [18] describes an ingenious re-entrant coupling mechanism. More recently, Shelton [19] has reported some novel strip-line configurations.

It can be shown that the even-mode impedance of coupled TEM-mode transmission lines is always greater than the odd-mode impedance. Because of this physical requirement, the realizability of the stepped-impedance prototype filter does not guarantee the realizability of the coupler. To insure realizability of the coupler, the impedances of the stepped-impedance prototype filter must all be greater than (or all less than) the terminating impedance. The restrictions on the insertion-loss polynomial that would guarantee realizability of the coupler are, as yet, unknown. However, the requirement that the normalized even-mode impedances of the coupler be greater than unity is satisfied in all the numerical solutions presented in this paper.

E. On −8.34-dB Directional Couplers

Recently, Shelton, Wolfe, and Van Wagoner [19] presented design data and demonstrated practical techniques of constructing −3.01-dB directional couplers by connecting two −8.34-dB couplers in tandem in the appropriate fashion. A −8.34-dB coupler is considerably easier to construct than a −3.01-dB coupler, and, therefore, the method of Shelton, Wolfe, and Van Wagoner [19] is of practical value. It is for this reason that the −8.34-dB coupler designs are included in the design tables.

VI. Conclusions

Design tables of optimum symmetrical TEM-mode coupled-transmission-line directional couplers of three, five, seven, and nine sections were presented. The designs give the maximum bandwidth for symmetrical couplers of a given number of sections, given mean coupling, and given coupling tolerance. To complete the tables, designs of maximally-flat directional couplers were also presented. An iterative method for determining new equal-ripple polynomials was presented, and a few examples of equal-ripple polynomials that approximate unity on the interval zero to one were given.

VII. Acknowledgment

The computer programs for obtaining equal-ripple and maximally-flat polynomials, and for the synthesis of the symmetrical couplers, were written by W. Wiebenson, P. Omlor and J. Ulrich, and Miss Elizabeth Tessman.

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Appendix

Tables of Parameters for Symmetrical TEM-Mode Coupled-Transmission-Line Directional Couplers

Table A-1

EVEN-MODE IMPEDANCES FOR EQUAL-RIPPLE SYMMETRICAL −3.01-db COUPLERS OF THREE SECTIONS

$(Z_{4-i} = Z_i)$

δ	Z_1	Z_2	w	B
0.05	1.14888	3.16095	0.86101	2.51187
0.10	1.17135	3.25984	1.00760	3.03063
0.15	1.19039	3.34049	1.10168	3.45275
0.20	1.20776	3.41242	1.17199	3.83085
0.25	1.22415	3.47932	1.22844	4.18429
0.30	1.23992	3.54311	1.27572	4.52271
0.35	1.25528	3.60495	1.31645	4.85178
0.40	1.27036	3.66560	1.35225	5.17521
0.45	1.28527	3.72563	1.38420	5.49559
0.50	1.30008	3.78546	1.41305	5.81489
0.60	1.32964	3.90585	1.46353	6.45616
0.70	1.35942	4.02894	1.50670	7.10860
0.80	1.38970	4.15648	1.54440	7.77966
0.90	1.42073	4.29005	1.57788	8.47591
1.00	1.45274	4.43120	1.60798	9.20361

Table A-2

EVEN-MODE IMPEDANCES FOR EQUAL-RIPPLE SYMMETRICAL −6-db COUPLERS OF THREE SECTIONS

$(Z_{4-i} = Z_i)$

δ	Z_1	Z_2	w	B
0.10	1.10298	2.09445	0.91996	2.70356
0.20	1.12090	2.14693	1.07404	3.31984
0.30	1.13625	2.18999	1.17223	3.83226
0.40	1.15038	2.22865	1.24518	4.29931
0.50	1.16385	2.26488	1.30345	4.74258
0.60	1.17680	2.29968	1.35201	5.17291
0.70	1.18952	2.33366	1.39364	5.59673
0.80	1.20208	2.36724	1.43006	6.01830
0.90	1.21454	2.40072	1.46241	6.44068
1.00	1.22698	2.43431	1.49150	6.86621

Table A-3

EVEN-MODE IMPEDANCES FOR EQUAL-RIPPLE SYMMETRICAL −8.34-db COUPLERS OF THREE SECTIONS

$(Z_{4-i} = Z_i)$

δ	Z_1	Z_2	w	B
0.05	1.06661	1.69824	0.76021	2.22636
0.10	1.07434	1.71858	0.89286	2.61290
0.15	1.08073	1.73468	0.97882	2.91703
0.20	1.08644	1.74864	1.04355	3.18211
0.25	1.09171	1.76127	1.09583	3.42397
0.30	1.09670	1.77299	1.13986	3.65041
0.35	1.10146	1.78405	1.17796	3.86595
0.40	1.10606	1.79461	1.21159	4.07347
0.45	1.11054	1.80478	1.24170	4.27495
0.50	1.11490	1.81463	1.26898	4.47178
0.55	1.11918	1.82424	1.29391	4.66502
0.60	1.12339	1.83365	1.31688	4.85550
0.65	1.12754	1.84289	1.33817	5.04386
0.70	1.13164	1.85200	1.35801	5.23063
0.75	1.13570	1.86101	1.37658	5.41623
0.80	1.13973	1.86993	1.39403	5.60103
0.85	1.14373	1.87878	1.41049	5.78534
0.90	1.14770	1.88759	1.42607	5.96943
0.95	1.15166	1.89636	1.44084	6.15354
1.00	1.15560	1.90510	1.45488	6.33787

Table A-4

EVEN-MODE IMPEDANCES FOR EQUAL-RIPPLE SYMMETRICAL −10-db COUPLERS OF THREE SECTIONS

$(Z_{4-i} = Z_i)$

δ	Z_1	Z_2	w	B
0.20	1.06945	1.57423	1.03140	3.12968
0.40	1.08475	1.60708	1.19816	3.98852
0.60	1.09817	1.63470	1.30282	4.73738
0.80	1.11075	1.66014	1.37959	5.44739
1.00	1.12290	1.68458	1.44020	6.14545

ALSO SEE PREVIOUS PAPER

Table A-5

EVEN-MODE IMPEDANCES FOR EQUAL-RIPPLE SYMMETRICAL
−20-db COUPLERS OF THREE SECTIONS
($Z_{4-i} = Z_i$)

δ	Z_1	Z_2	w	B
0.20	1.02070	1.14914	1.00980	3.03958
0.40	1.02497	1.15617	1.17423	3.84396
0.60	1.02866	1.16197	1.27772	4.53804
0.80	1.03208	1.16720	1.35381	5.19011
1.00	1.03534	1.17213	1.41398	5.82570

Table A-6

EVEN-MODE IMPEDANCES FOR EQUAL-RIPPLE SYMMETRICAL
−3 01-db COUPLERS OF FIVE SECTIONS
($Z_{6-i} = Z_i$)

δ	Z_1	Z_2	Z_3	w	B
0.05	1.05972	1.32624	3.81243	1.20488	4.03071
0.10	1.07851	1.37268	3.97615	1.32559	4.93114
0.15	1.09451	1.40890	4.10191	1.39889	5.65437
0.20	1.10921	1.44029	4.21023	1.45184	6.29714
0.25	1.12314	1.46883	4.30864	1.49333	6.89474
0.30	1.13659	1.49551	4.40089	1.52744	7.46462
0.35	1.14973	1.52091	4.48917	1.55639	8.01698
0.40	1.16266	1.54541	4.57491	1.58152	8.55845
0.45	1.17547	1.56926	4.65912	1.60371	9.09367
0.50	1.18822	1.59265	4.74253	1.62357	9.62609
0.60	1.21370	1.63864	4.90924	1.65791	10.69292
0.70	1.23941	1.68425	5.07867	1.68691	11.77568
0.80	1.26555	1.73013	5.25363	1.71196	12.88720
0.90	1.29235	1.77678	5.43655	1.73402	14.03860
1.00	1.31998	1.82466	5.62978	1.75370	15.24047

Table A-7

EVEN-MODE IMPEDANCES FOR EQUAL-RIPPLE SYMMETRICAL
−6-db COUPLERS OF FIVE SECTIONS
($Z_{6-i} = Z_i$)

δ	Z_1	Z_2	Z_3	w	B
0.10	1.04501	1.21972	2.38181	1.25446	4.34522
0.20	1.06052	1.25302	2.46010	1.37766	5.49738
0.30	1.07392	1.27919	2.52068	1.45202	6.29953
0.40	1.08633	1.30203	2.57332	1.50548	7.08866
0.50	1.09818	1.32294	2.62159	1.54720	7.83386
0.60	1.10969	1.34262	2.66727	1.58135	8.55462
0.70	1.12099	1.36146	2.71142	1.61023	9.26242
0.80	1.13217	1.37976	2.75470	1.63529	9.96482
0.90	1.14328	1.39772	2.79760	1.65716	10.66721
1.00	1.15438	1.41542	2.84048	1.67673	11.37370

Table A-8

EVEN-MODE IMPEDANCES FOR EQUAL-RIPPLE SYMMETRICAL
−8.34-db COUPLERS OF FIVE SECTIONS
($Z_{6-i} = Z_i$)

δ	Z_1	Z_2	Z_3	w	B
0.05	1.02538	1.14102	1.85802	1.11764	3.53328
0.10	1.03211	1.15690	1.89019	1.23184	4.20727
0.15	1.03770	1.16899	1.91418	1.30256	4.73524
0.20	1.04271	1.17918	1.93414	1.35395	5.19150
0.25	1.04737	1.18822	1.95170	1.39442	5.60527
0.30	1.05179	1.19648	1.96764	1.42783	5.99090
0.35	1.05602	1.20417	1.98243	1.45627	6.35662
0.40	1.06012	1.21142	1.99635	1.48104	6.70767
0.45	1.06412	1.21833	2.00960	1.50296	7.04761
0.50	1.06803	1.22497	2.02232	1.52262	7.37898
0.55	1.07187	1.23138	2.03462	1.54043	7.70370
0.60	1.07565	1.23760	2.04658	1.55670	8.02323
0.65	1.07939	1.24367	2.05826	1.57168	8.33872
0.70	1.08309	1.24960	2.06971	1.58554	8.65112
0.75	1.08675	1.25542	2.08098	1.59844	8.96119
0.80	1.09039	1.26114	2.09210	1.61050	9.26959
0.85	1.09401	1.26678	2.10310	1.62182	9.57685
0.90	1.09761	1.27235	2.11401	1.63247	9.88347
0.95	1.10119	1.27785	2.12484	1.64253	10.18985
1.00	1.10476	1.28331	2.13562	1.65206	10.49635

Table A-9

EVEN-MODE IMPEDANCES FOR EQUAL-RIPPLE SYMMETRICAL
−10-db COUPLERS OF FIVE SECTIONS
($Z_{6-i} = Z_i$)

δ	Z_1	Z_2	Z_3	w	B
0.20	1.03418	1.14316	1.70922	1.34442	5.10148
0.40	1.04784	1.16808	1.75305	1.47118	6.56407
0.60	1.05996	1.18815	1.78805	1.54675	7.82513
0.80	1.07140	1.20606	1.81943	1.60053	9.01322
1.00	1.08249	1.22280	1.84912	1.64210	10.17639

Table A-10

EVEN-MODE IMPEDANCES FOR EQUAL-RIPPLE SYMMETRICAL −20-db COUPLERS OF FIVE SECTIONS

$(Z_{6-i} = Z_i)$

δ	Z_1	Z_2	Z_3	w	B
0.20	1.01016	1.04183	1.17873	1.32734	4.94656
0.40	1.01406	1.04855	1.18767	1.45350	6.31936
0.60	1.01747	1.05386	1.19463	1.52888	7.49038
0.80	1.02066	1.05851	1.20073	1.58261	8.58338
1.00	1.02371	1.06280	1.20638	1.62420	9.64410

Table A-11

EVEN-MODE IMPEDANCES FOR EQUAL-RIPPLE SYMMETRICAL −3.01-db COUPLERS OF SEVEN SECTIONS

$(Z_{8-i} = Z_i)$

δ	Z_1	Z_2	Z_3	Z_4	w	B
0.05	1.03635	1.14905	1.50280	4.39954	1.40024	5.6693
0.10	1.05240	1.18406	1.56753	4.61180	1.49705	6.9531
0.15	1.06643	1.21168	1.61640	4.77112	1.55447	7.9780
0.20	1.07950	1.23581	1.65795	4.90662	1.59539	8.8860
0.25	1.09201	1.25786	1.69523	5.02872	1.62715	9.7283
0.30	1.10419	1.27860	1.72975	5.14254	1.65308	10.5302
0.35	1.11615	1.29840	1.76238	5.25103	1.67497	11.3064
0.40	1.12798	1.31754	1.79367	5.35611	1.69388	12.0666
0.45	1.13975	1.33622	1.82400	5.45909	1.71051	12.8174
0.50	1.15149	1.35457	1.85365	5.56097	1.72534	13.5637
0.60	1.17505	1.39069	1.91172	5.76434	1.75090	15.0578
0.70	1.19890	1.42654	1.96915	5.97094	1.77238	16.5728
0.80	1.22323	1.46258	2.02682	6.18437	1.79087	18.1270
0.90	1.24820	1.49918	2.08545	6.40775	1.80710	19.7360
1.00	1.27399	1.53668	2.14566	6.64407	1.82155	21.4147

Table A-12

EVEN-MODE IMPEDANCES FOR EQUAL-RIPPLE SYMMETRICAL −6-db COUPLERS OF SEVEN SECTIONS

$(Z_{8-i} = Z_i)$

δ	Z_1	Z_2	Z_3	Z_4	w	B
0.10	1.02686	1.10756	1.32930	2.62516	1.44052	6.1494
0.20	1.04246	1.13419	1.37278	2.72038	1.53802	7.6583
0.30	1.05449	1.15540	1.40584	2.79246	1.59558	8.8908
0.40	1.06580	1.17408	1.43416	2.85438	1.63645	10.0026
0.50	1.07670	1.19128	1.45977	2.91078	1.66806	11.0505
0.60	1.08735	1.20755	1.48367	2.96391	1.69378	12.0626
0.70	1.09787	1.22318	1.50642	3.01511	1.71541	13.0553
0.80	1.10831	1.23839	1.52841	3.06523	1.73404	14.0396
0.90	1.11872	1.25331	1.54989	3.11486	1.75036	15.0232
1.00	1.12915	1.26805	1.57104	3.16446	1.76487	16.0119

Table A-13

EVEN-MODE IMPEDANCES FOR EQUAL-RIPPLE SYMMETRICAL −8.34-db COUPLERS OF SEVEN SECTIONS

$(Z_{8-i} = Z_i)$

δ	Z_1	Z_2	Z_3	Z_4	w	B
0.05	1.01460	1.06403	1.21141	1.99183	1.32568	4.9319
0.10	1.02033	1.07694	1.23301	2.03194	1.42127	5.9117
0.15	1.02519	1.08680	1.24872	2.06076	1.47818	6.6655
0.20	1.02963	1.09518	1.26167	2.08436	1.51889	7.3140
0.25	1.03379	1.10266	1.27297	2.10489	1.55059	7.9005
0.30	1.03778	1.10953	1.28316	2.12339	1.57653	8.4458
0.35	1.04163	1.11595	1.29256	2.14044	1.59848	8.9622
0.40	1.04538	1.12204	1.30136	2.15641	1.61749	9.4572
0.45	1.04905	1.12786	1.30969	2.17156	1.63423	9.9360
0.50	1.05265	1.13346	1.31764	2.18606	1.64919	10.4022
0.55	1.05621	1.13889	1.32528	2.20004	1.66270	10.8588
0.60	1.05972	1.14417	1.33267	2.21361	1.67500	11.3077
0.65	1.06320	1.14933	1.33984	2.22683	1.68629	11.7507
0.70	1.06666	1.15438	1.34684	2.23979	1.69672	12.1891
0.75	1.07009	1.15934	1.35368	2.25251	1.70640	12.6240
0.80	1.07350	1.16423	1.36040	2.26506	1.71543	13.0563
0.85	1.07689	1.16904	1.36700	2.27746	1.72389	13.4870
0.90	1.08028	1.17381	1.37351	2.28975	1.73184	13.9165
0.95	1.08365	1.17852	1.37993	2.30194	1.73934	14.3456
1.00	1.08702	1.18319	1.38629	2.31408	1.74643	14.7747

Table A-14

EVEN-MODE IMPEDANCES FOR EQUAL-RIPPLE SYMMETRICAL −10-db COUPLERS OF SEVEN SECTIONS

$(Z_{8-i} = Z_i)$

δ	Z_1	Z_2	Z_3	Z_4	w	B
0.20	1.02360	1.07622	1.20802	1.81699	1.51198	7.1965
0.40	1.03597	1.09725	1.23839	1.86715	1.61028	9.2638
0.60	1.04718	1.11444	1.26213	1.90649	1.66773	11.0383
0.80	1.05788	1.12991	1.28298	1.94149	1.70815	12.7059
1.00	1.06834	1.14444	1.30229	1.97446	1.73917	14.3359

Table A-15

EVEN-MODE IMPEDANCES FOR EQUAL-RIPPLE SYMMETRICAL
−20-db COUPLERS OF SEVEN SECTIONS
($Z_{8-i} = Z_i$)

δ	Z_1	Z_2	Z_3	Z_4	w	B
0.20	1.00697	1.02256	1.05976	1.20128	1.49853	6.9766
0.40	1.01052	1.02846	1.06767	1.21112	1.59672	8.9188
0.60	1.01369	1.03320	1.07372	1.21863	1.65421	10.5678
0.80	1.01669	1.03740	1.07894	1.22515	1.69472	12.1029
1.00	1.01958	1.04129	1.08368	1.23116	1.72584	13.5903

Table A-16

EVEN-MODE IMPEDANCES FOR EQUAL-RIPPLE SYMMETRICAL
−3.01-db COUPLERS OF NINE SECTIONS
($Z_{10-i} = Z_i$)

δ	Z_1	Z_2	Z_3	Z_4	Z_5	w	B
0.05	1.02680	1.09163	1.24706	1.66958	4.93133	1.5218	7.365
0.10	1.04112	1.12024	1.29488	1.74863	5.18240	1.6012	9.030
0.15	1.05391	1.14328	1.33137	1.80742	5.36886	1.6478	10.356
0.20	1.06598	1.16366	1.36260	1.85696	5.52654	1.6807	11.528
0.25	1.07763	1.18248	1.39075	1.90116	5.66814	1.7062	12.615
0.30	1.08904	1.20027	1.41691	1.94192	5.79985	1.7269	13.649
0.35	1.10030	1.21737	1.44168	1.98035	5.92523	1.7444	14.648
0.40	1.11149	1.23397	1.46548	2.01711	6.04655	1.7594	15.627
0.45	1.12264	1.25023	1.48856	2.05271	6.16540	1.7726	16.594
0.50	1.13379	1.26625	1.51114	2.08747	6.28296	1.7844	17.554
0.60	1.15624	1.29789	1.55536	2.15551	6.51769	1.8046	19.475
0.70	1.17904	1.32941	1.59902	2.22278	6.75634	1.8216	21.423
0.80	1.20234	1.36117	1.64277	2.29038	7.00316	1.8362	23.421
0.90	1.22630	1.39348	1.68712	2.35918	7.26188	1.8490	25.488
1.00	1.25107	1.42660	1.73250	2.42995	7.53602	1.8604	27.644

Table A-17

EVEN-MODE IMPEDANCES FOR EQUAL-RIPPLE SYMMETRICAL
−6-db COUPLERS OF NINE SECTIONS
($Z_{10-i} = Z_i$)

δ	Z_1	Z_2	Z_3	Z_4	Z_5	w	B
0.10	1.02201	1.06888	1.17282	1.42807	2.83542	1.5550	7.989
0.20	1.03437	1.09137	1.20736	1.47877	2.94305	1.6345	9.943
0.30	1.04554	1.10967	1.23393	1.51676	3.02373	1.6809	11.535
0.40	1.05615	1.12599	1.25686	1.54902	3.09269	1.7136	12.969
0.50	1.06645	1.14117	1.27768	1.57805	3.15533	1.7389	14.319
0.60	1.07658	1.15561	1.29716	1.60504	3.21427	1.7594	15.622
0.70	1.08662	1.16957	1.31574	1.63070	3.27103	1.7765	16.900
0.80	1.09661	1.18320	1.33370	1.65546	3.32658	1.7913	18.166
0.90	1.10661	1.19661	1.35125	1.67962	3.38161	1.8042	19.431
1.00	1.11663	1.20989	1.36852	1.70342	3.43663	1.8157	20.702

Table A-18

EVEN-MODE IMPEDANCES FOR EQUAL-RIPPLE SYMMETRICAL
−8.34-db COUPLERS OF NINE SECTIONS
($Z_{10-i} = Z_i$)

δ	Z_1	Z_2	Z_3	Z_4	Z_5	w	B
0.05	1.01032	1.03838	1.10598	1.27508	2.10668	1.4599	6.406
0.10	1.01536	1.04904	1.12341	1.30048	2.15200	1.5392	7.681
0.15	1.01974	1.05735	1.13622	1.31862	2.18413	1.5858	8.658
0.20	1.02379	1.06452	1.14687	1.33341	2.21025	1.6190	9.498
0.25	1.02764	1.07099	1.15622	1.34622	2.23285	1.6446	10.256
0.30	1.03134	1.07697	1.16469	1.35771	2.25315	1.6656	10.960
0.35	1.03494	1.08261	1.17253	1.36825	2.27180	1.6832	11.627
0.40	1.03846	1.08798	1.17989	1.37809	2.28925	1.6985	12.265
0.45	1.04193	1.09314	1.18687	1.38738	2.30577	1.7119	12.883
0.50	1.04534	1.09813	1.19356	1.39622	2.32156	1.7238	13.484
0.55	1.04872	1.10298	1.19999	1.40471	2.33677	1.7346	14.072
0.60	1.05206	1.10771	1.20622	1.41290	2.35152	1.7444	14.650
0.65	1.05538	1.11234	1.21228	1.42085	2.36589	1.7534	15.221
0.70	1.05868	1.11689	1.21819	1.42858	2.37996	1.7617	15.785
0.75	1.06196	1.12137	1.22398	1.43615	2.39378	1.7694	16.345
0.80	1.06523	1.12579	1.22966	1.44356	2.40740	1.7766	16.901
0.85	1.06849	1.13016	1.23525	1.45084	2.42087	1.7833	17.455
0.90	1.07174	1.13448	1.24076	1.45802	2.43421	1.7896	18.008
0.95	1.07498	1.13877	1.24620	1.46511	2.44745	1.7955	18.560
1.00	1.07823	1.14302	1.25158	1.47211	2.46063	1.8011	19.111

Table A-19

EVEN-MODE IMPEDANCES FOR EQUAL-RIPPLE SYMMETRICAL
−10-db COUPLERS OF NINE SECTIONS
($Z_{10-i} = Z_i$)

δ	Z_1	Z_2	Z_3	Z_4	Z_5	w	B
0.20	1.01889	1.05161	1.11743	1.26387	1.90628	1.6133	9.345
0.40	1.03041	1.07004	1.14313	1.29777	1.96074	1.6927	12.016
0.60	1.04103	1.08543	1.16344	1.32390	2.00313	1.7386	14.303
0.80	1.05127	1.09945	1.18139	1.34672	2.04073	1.7708	16.450
1.00	1.06133	1.11271	1.19805	1.36779	2.07614	1.7954	18.547

Table A-20

EVEN-MODE IMPEDANCES FOR EQUAL-RIPPLE SYMMETRICAL
−20-db COUPLERS OF NINE SECTIONS
($Z_{10-i} = Z_i$)

δ	Z_1	Z_2	Z_3	Z_4	Z_5	w	B
0.20	1.00555	1.01529	1.03447	1.07471	1.21931	1.6024	9.061
0.40	1.00886	1.02054	1.04153	1.08328	1.22965	1.6818	11.571
0.60	1.01187	1.02485	1.04700	1.08974	1.23748	1.7278	13.697
0.80	1.01474	1.02871	1.05175	1.09527	1.24426	1.7601	15.674
1.00	1.01753	1.03232	1.05608	1.10028	1.25049	1.7848	17.588

Table A-21

EVEN-MODE IMPEDANCES FOR MAXIMALLY FLAT SYMMETRICAL COUPLERS OF THREE SECTIONS

$(Z_{4-i} = Z_i)$

-3.01 db Coupling

Z_1	Z_2	w	B
1.10410	2.94302	1.46466	6.47184

-6 db Coupling

Z_1	Z_2	w	B
1.06810	1.97911	1.37435	5.39334

-10 db Coupling

Z_1	Z_2	w	B
1.04110	1.50382	1.33123	4.98113

-20 db Coupling

Z_1	Z_2	w	B
1.01260	1.13356	1.30852	4.78465

Table A-22

EVEN-MODE IMPEDANCES FOR MAXIMALLY FLAT SYMMETRICAL COUPLERS OF FIVE SECTIONS

$(Z_{6-i} = Z_i)$

-3.01 db Coupling

Z_1	Z_2	Z_3	w	B
1.01634	1.18837	3.28771	1.56911	8.28314

-6 db Coupling

Z_1	Z_2	Z_3	w	B
1.01231	1.12196	2.13095	1.49525	6.92473

-10 db Coupling

Z_1	Z_2	Z_3	w	B
1.00755	1.07296	1.57346	1.45978	6.40442

-20 db Coupling

Z_1	Z_2	Z_3	w	B
1.00235	1.02216	1.14966	1.44104	6.15614

Table A-23

EVEN-MODE IMPEDANCES FOR MAXIMALLY FLAT SYMMETRICAL COUPLERS OF SEVEN SECTIONS

$(Z_{8-i} = Z_i)$

-3.01 db Coupling

Z_1	Z_2	Z_3	Z_4	w	B
1.00375	1.04173	1.25855	3.55017	1.62962	9.7996

-6 db Coupling

Z_1	Z_2	Z_3	Z_4	w	B
1.00254	1.02791	1.16589	2.24311	1.56569	8.2100

-10 db Coupling

Z_1	Z_2	Z_3	Z_4	w	B
1.00157	1.01708	1.09855	1.62369	1.53490	7.6003

-20 db Coupling

Z_1	Z_2	Z_3	Z_4	w	B
1.00049	1.00530	1.02968	1.16096	1.51861	7.3092

Table A-24

EVEN-MODE IMPEDANCES FOR MAXIMALLY FLAT SYMMETRICAL COUPLERS OF NINE SECTIONS

$(Z_{10-i} = Z_i)$

-3.01 db Coupling

Z_1	Z_2	Z_3	Z_4	Z_5	w	B
1.00081	1.01035	1.06619	1.31891	3.76506	1.6703	11.131

-6 db Coupling

Z_1	Z_2	Z_3	Z_4	Z_5	w	B
1.00055	1.00700	1.04409	1.20303	2.33294	1.6131	9.338

-10 db Coupling

Z_1	Z_2	Z_3	Z_4	Z_5	w	B
1.00034	1.00431	1.02689	1.11989	1.66324	1.5855	8.651

-20 db Coupling

Z_1	Z_2	Z_3	Z_4	Z_5	w	B
1.00011	1.00135	1.00831	1.03587	1.16969	1.5709	8.322